Geostatistical Approach to Understanding the Effect of Rainfall Spatial-Temporal Uncertainty on a Small Urban Hydraulic Model

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Highlights

- A novel approach to calculate spatial-temporal uncertainty of rain gauge data
- Consideration of the effects of uncertainty in rain gauge data on a hydrodynamic sewer network model of a small urban catchment

Introduction

Rainfall is an imperative parameter in hydrological modelling studies due to the nature of the driving runoff processes (see Ballinas-Gonzalez *et al*., 2020), However, it can be highly variable in both time and space, which can cause considerable uncertainty (see Fraga *et al.,* 2019). Ochoa-Rodriguez *et al*. (2015) showed this to be a particular problem in small urban catchments, where short temporal averaging intervals are often of interest. Common practice in hydrological models is to use areal average rainfall intensity (AARI) as input. Common practice in urban drainage network models, is to overlay a Thiessens polygon with the network model, and use this to allocate different rain gauges to groups of subcatchments. Thus point observations from rain gauges are assumed to be valid for cover larger areas of land (Muthusamy *et al*., 2017). This is a deterministic approach, which does not provide information into how spatially variable rainfall is within these groups of subcatchments. Alternatively, geo-statistical methods would offer a measure of predictive error, providing information on uncertainty associated with predicted rainfall over a subcatchment. Muthusamy et al. (2017) presented a geo-statistical approach to derive AARI and associated uncertainty from observed rain gauge data. This study aims to look at the implication of this measured uncertainty in the AARI when applying to a small urban network model, and the effects on peak flow.

Methodology

Pooling of Sample Variograms

It is typical in the UK to follow the CIWEM UDG hydraulic modelling guide (2016), having at least 1+1 gauges per km^2, and thus recommending a few tens of gauges for a 'medium' size town. As at any given time with the number of locations in the rain gauge network, there would be fewer and smaller spatial lags compared to that normally used in geo-statistical modelling. Therefore, this study utilised the method developed by Muthusamy et al. (2017) to increases the number of pairs by pooling sample variograms. With *t* time instances and *n* measurements locations the pooling technique creates $t \times 1/2 \times n \times (n-1)$ spatial pairs. Only sample variograms with similar rainfall characteristics would be pooled as this procedure has the underlying assumption that the spatial variability over the pooled time instants is constant. Three subclasses of varying rainfall intensity, <5 mm h^{-1}, 5-10 mm h^{-1} and >10 mm hr^{-1}, were derived. Observations within these intensity classes were standardised, using the mean and standard deviation. Data needs to be normally distributed prior to calibration of a geo-statistical model; the normal score transformation is an established method to transform a variable distribution to a Gaussian distribution (see Bogner *et al*., 2012).

Spatial Stochastic Simulation

For a geo-statistical model of rainfall intensity at any location with limited data and small catchment size the trend is assumed to be constant. This assumption allows an ordinary kriging system to be appropriate for solving spatial interpolation (Isaaks and Srivastava, 1989). Ordinary kriging weights are calculated, which allows point rainfall intensities to be predicted using point kriging at any given point by taking a weighted average of the observed rainfall intensities. To calculate the average rainfall intensity over the catchment, a spatial stochastic simulation approach was chosen. The output of spatial stochastic simulation is a set of 500 possible realisations, the mean of which approximates the kriging standard deviation. These realisations are then back transformed by applying an inverse Gaussian cumulative distribution and then spatially averaged. For the values derived from the spatial stochastic simulation outside the transformed data range, linear extrapolation was used. This set of data is a simple random sample of the catchment average rainfall. Finally, an inverse standardisation of the mean and standard deviation of this distribution is completed to derive the AARI and associated uncertainty measure (standard deviation). The geostatistical model was applied to rainfall data collected from a network of 8 rain gauge locations, where 2 gauges were paired in each location situated in an area of approximately 500m by 1400m.

Urban Hydraulic Model

Rainfall estimates from the AARI and associated uncertainty, were applied to a hydraulic network model of a small urban catchment which was approx. 60km away from the area which the rainfall was collected and was believed to have similar climatic characteristics. The network model simulated three different rainfall events, with varying intensities. Peak flow rates were simulated in 5 pipes located at approximately 100, 500, 1000, 1500 and 2500 m from the most upstream end of the network (referred to as link 1, 2, 3, 4 & 5 respectively), for different temporal resolution and at the upper and lower 95% interval of the AARI for all three rainfall events.

Results and discussion

While short time intervals are of greater interest in urban hydrology, they can also lead to large uncertainties, Figure 1. This is a result of the combination of higher spatial variability and larger tipping bucket error at smaller temporal averaging intervals. When the averaging interval is larger than 15 min the prediction interval width becomes negligible. This agrees with previous studies in the literature, who suggest the effect is of a results of improvement in measurement accuracy and spatial correlation (Villarini et al., 2008). Figure 2 illustrates the effect of the 95% prediction intervals on the range of peak pipe flows which could all be expected as true. The 2 and 5 minute interval show larger differences in peak flow, due to the larger 95% prediction intervals and the 2, 5 and 15

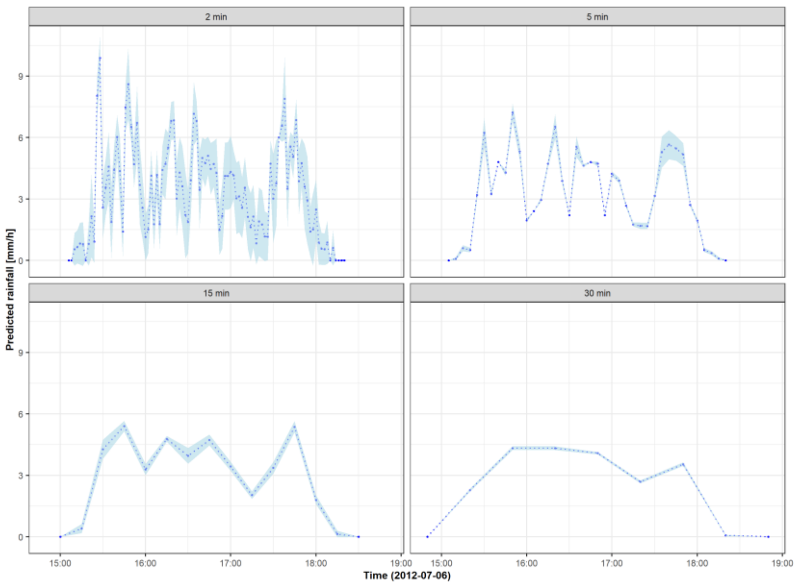

Figure 1*; Predictions of AARI (indicated by points) together with 95 % prediction intervals (indicated by blue zone) for rainfall event 7*

minute intervals often overlap (Figure 1 and 2). For pipes 2, 3 & 4 for the 2 & 5-minute intervals, the 95% rainfall prediction interval leads to maximum +37% and -32% changes in predicted peak in-pipe flows. For comparison CIWEM (2017) recommends a 'green' confidence score when simulated peak pipe flow is between +25% and -10% of observed peak flow.

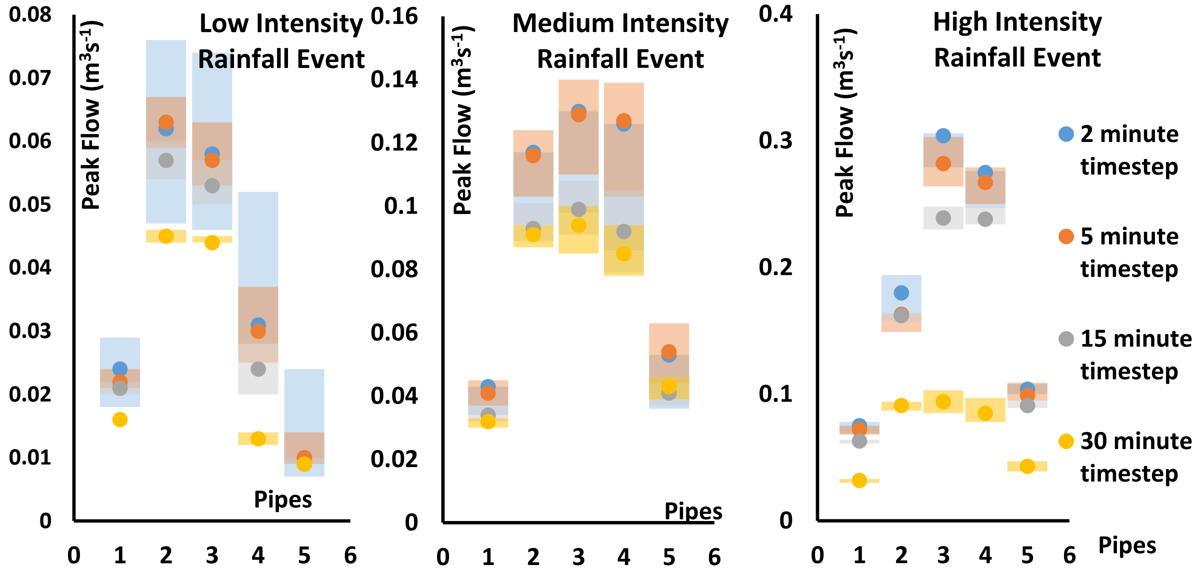

Figure 2*; Peak flow at five links for 3 different rainfall events at 2, 5, 15 & 30 minutes. Point shows the predicted rainfall and shaded area is range from % predicted interval.*

Conclusions and future work

Currently it is common practice to ignore the uncertainty associated with the data from rain gauges in the calibration or validation of a hydraulic model build. However, results show that whether a model passes industrial criteria of verification could potentially be impacted by the spatial scale selected to collect the data. A model may be altered to pass calibration and validation without proper understanding of where the difference in observed and modelled flows came from. Future work aims to use Monte Carlo simulation to identify a probabilistic based way of quantitatively incorporating the spatial-temporal uncertainty around rain gauge data into urban hydraulic models.

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